

Exam 3

08 July 2020

1. let $u = 2x^2 \Rightarrow \frac{du}{dx} = 4x \Rightarrow dx = \frac{du}{4x}$

$$\int \frac{x \cos u \cdot x du}{4x} = \frac{1}{4} \int \cos u du$$

$$= \frac{\sin u}{4} + C_1$$

$$= \boxed{\frac{\sin(2x^2)}{4} + C}$$

-5

2. $\int 12(4x-3)^{-5} dx$: let $u = 4x-3$

$$\frac{du}{dx} = 4 \Rightarrow dx = \frac{du}{4}$$

$$\int 3 u^{-5} du = \frac{3 u^{-6}}{-6} = -\frac{u^{-6}}{2} + C_1$$

$$= \boxed{-\frac{(4x-3)^{-6}}{2} + C}$$

3. $\int x^5 \sqrt{x^6+7} dx$ let $u = x^6+7$

$$\frac{du}{dx} = x^5 \Rightarrow dx = \frac{du}{x^5}$$

$$\int u^{1/2} du = \frac{2}{3} u^{3/2}$$

$$= \boxed{\frac{2}{3} (x^6+7)^{3/2} + C}$$

$$4. \int \sin(6x-5) dx \quad : \text{ Let } u = 6x-5 \quad \frac{du}{dx} = 6$$

$$dx = \frac{du}{6}$$

$$= \frac{1}{6} \int \sin u \, du = -\frac{1}{6} \cos u$$

$$= \boxed{-\frac{1}{6} \cos(6x-5) + C}$$

$$5. \int \csc^2(6\theta+8) d\theta \quad : \text{ Let } u = 6\theta+8$$

$$\frac{du}{d\theta} = 6, \quad d\theta = \frac{du}{6}$$

$$\int \frac{\csc^2 u}{6} du = \frac{1}{6} \int \csc^2 u$$

$$= -\frac{1}{6} \cot u = \boxed{-\frac{1}{6} \cot(6\theta+8) + C}$$

$$6. \int \frac{\cos x}{1+2\sin x} dx = \frac{1}{2} \ln |1+2\sin x|$$

$$\text{let } u = 1+2\sin x \Rightarrow \frac{du}{dx} = 2\cos x \Rightarrow dx = \frac{du}{2\cos x}$$

$$\int \frac{\cos x}{u} \times \frac{du}{2\cos x} = \frac{1}{2} \int \frac{du}{u} = \frac{1}{2} \ln u$$

$$= \boxed{\frac{1}{2} \ln |1+2\sin x| + C}$$

$$7. \int 3e^{-7x} dx = \boxed{-\frac{3}{7}e^{-7x} + C} \quad \text{using formula}$$

$$8. \int \frac{dx}{\sqrt{16-x^2}} \Rightarrow \text{let } u = \frac{x}{4}$$

$$u = \frac{x}{4}, \quad \frac{du}{dx} = \frac{1}{4} \quad dx = 4 du$$

$$\int \frac{4}{\sqrt{16-16u^2}} du = \int \frac{1}{\sqrt{1-u^2}}$$

$$= \arcsin u$$

$$= \boxed{\arcsin\left(\frac{x}{4}\right) + C}$$

~~$$9. \int \frac{6x^2 dx}{(8+x^6)^{3/2}} \Rightarrow \text{let } u = \frac{x^3}{2^{3/2}} \Rightarrow \frac{du}{dx} = \frac{3x^2}{2^{3/2}}$$~~

~~$$dx = \frac{2^{3/2}}{3x^2} du$$~~

~~$$\int \frac{2^{5/2}}{8u^2+8} du = \frac{1}{\sqrt{2}} \int \frac{1}{u^2+1} du$$~~

~~$$= \frac{1}{\sqrt{2}} \arctan u$$~~

~~$$= \frac{1}{\sqrt{2}} \arctan \frac{x^3}{2^{3/2}} + C$$~~

9. Let $u = 8 + x^6 \Rightarrow \frac{du}{dx} = x^5 \quad dx = \frac{du}{x^5}$

$$\int \frac{6x^5}{u^6} \times \frac{du}{x^5} = \int \frac{6du}{u^6} = 6 \int u^{-6} du$$

$$= -\frac{6}{5} u^{-5}$$

$$= -\frac{6}{5} (8 + x^6)^{-5} + C$$

$$= \boxed{-\frac{6}{5 (8 + x^6)^5} + C}$$

10. $\int x^2 \sqrt{x^3 + 2} dx \Rightarrow$ Let $u = x^3 + 2$

$$\frac{du}{dx} = 3x^2 \Rightarrow dx = \frac{du}{3x^2}$$

$$\int \cancel{x^2} \sqrt{u} \frac{du}{3\cancel{x^2}} = \frac{1}{3} \int \sqrt{u} du$$

$$= \frac{1}{3} \times \frac{2}{3} u^{3/2}$$

$$= \boxed{\frac{2}{9} (x^3 + 2)^{3/2}}$$

$$11. \int \frac{\cos(\ln x - 8)}{x} dx \quad : \text{ Let } u = \ln x - 8$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$dx = x du$$

$$\int \frac{\cos u}{x} x du = \int \cos u du$$

$$= \sin u$$

$$= \boxed{\sin(\ln x - 8) + C}$$

$$12. \int \sec 6t dt, \text{ Let } u = 6t$$

$$\frac{du}{dt} = 6, dt = \frac{du}{6}$$

$$= \frac{1}{6} \int \sec u du \quad \text{using expansion } \tan u + \sec u$$

$$= \frac{1}{6} \int \frac{\sec u \tan u + \sec^2 u}{\tan u + \sec u} du$$

$$= \int \frac{1}{v} dv = \ln v$$

$$= \frac{1}{6} \ln(\tan(u) + \sec u)$$

$$= \boxed{\frac{1}{6} \ln|\tan 6t + \sec 6t| + C}$$

$$13. \int \csc \frac{t}{2} dt \quad ; \quad u = \frac{t}{2}, \quad dt = 2du$$

$$= 2 \int \csc(u) du$$

Expand using $\csc(u) + \cot(u)$

$$\int \frac{\csc^2 u + \cot u \csc u}{\csc u + \cot u} du$$

$$\text{Let } v = \csc u + \cot u \Rightarrow \frac{dv}{du} = -\csc u - \cot u \csc u$$

$$du = \frac{1}{-\csc^2(u) - \cot u \csc u} dv$$

$$= \int \frac{1}{v} dv = \ln v$$

$$= -2 \ln \csc(u) + \cot u$$

$$= \boxed{-2 \ln \csc\left(\frac{t}{2}\right) + \cot\left(\frac{t}{2}\right) + C}$$

$$14. \int \frac{5 dx}{1-2x} \Rightarrow \text{let } u = 1-2x, \quad \frac{du}{dx} = -2, \quad dx = -\frac{du}{2}$$

$$\int \frac{-5 du}{2u} = -\frac{5}{2} \int \frac{du}{u} = -\frac{5}{2} \ln u$$

$$= \boxed{-\frac{5}{2} \ln |1-2x| + C}$$

$$15. \int x^5 e^{x^6} dx \Rightarrow \text{Let } u = -x^6 \Rightarrow \frac{du}{dx} = -6x^5$$

$$dx = \frac{-1}{6x^5} du$$

$$= +\frac{1}{6} \int e^u du = -\frac{e^{-x^6}}{6} + C$$

$$16. \int 8^{\sin x} \cos x dx : \text{Let } u = \sin x, dx = \frac{1}{\cos x} du$$

$$= \int 8^u du = \frac{8^u}{\ln 8}$$

$$= \frac{8^{\sin x}}{\ln(8)} + C$$

$$17. \int x^4 \sqrt{x^5 + 10} dx : \text{Let } u = x^5 + 10$$

$$\frac{du}{dx} = 5x^4 \Rightarrow dx = \frac{du}{5x^4}$$

$$\int \cancel{x^4} \sqrt{u} \frac{du}{\cancel{5x^4}} = \int \sqrt{u} du = \frac{2}{3} u^{3/2}$$

$$= \frac{2}{3} (x^5 + 10)^{3/2} + C$$

$$18. \int x^5 dx = \frac{x^6}{6} + C$$

$$19. \int (8x^2 + 1) dx = \int 8x^2 dx + \int 1 dx$$

$$= \frac{8x^3}{3} + x + C$$

$$20. \int (4t^2 + 7t - 3) dt$$

$$= \frac{4t^3}{3} + \frac{7t^2}{2} - 3t + C$$

$$21. \int \frac{3\sqrt{x} - 5}{x^2} dx = \frac{-6}{\sqrt{x}} + \frac{5}{x} + C$$

$$22. \int 8e^{4y} dy = \frac{8e^{4y}}{4} + C$$

$$= 2e^{4y} + C$$

$$23. \int 5^{4x} dx \quad \text{let } u = 4x$$

$$\frac{du}{dx} = 4 \quad dx = \frac{du}{4}$$

$$\int \frac{1}{4} 5^u du$$

$$= \frac{1}{4} \int 5^u du = \frac{5^u}{4 \ln(5)}$$

$$= \boxed{\frac{5^{4x}}{4 \ln(5)} + C}$$

$$24. \int \left(\frac{1}{x} + \frac{2}{x^2} + \frac{3}{x^3} \right) dx$$

$$\int \frac{1}{x} dx + \int \frac{2}{x^2} dx + \int \frac{3}{x^3} dx$$

$$= \boxed{\ln|x| - \frac{2}{x} - \frac{3}{2x^2} + C}$$

$$25. \int \left(\frac{4}{x} + 2e^x \right) dx = 4 \ln|x| + 2e^x + C$$

$$\int \frac{4}{x} dx + \int 2e^x dx = \boxed{4 \ln x + 2e^x + C}$$

$$26. \int 4(2x+5)^3 dx$$

$$\text{let } u = 2x+5 \Rightarrow \frac{du}{dx} = 2 \Rightarrow dx = \frac{du}{2}$$

$$\int 4 u^3 \frac{du}{2} = \int 2 u^3 du$$

$$\frac{2 u^4}{4} + C_1 = \frac{1}{2} u^4 + C_1$$

$$\boxed{\frac{(2x+5)^4}{2} + C}$$

$$27. \int \frac{8 dy}{(y-9)^3} \quad : \quad u = y-9 \quad \frac{du}{dy} = 1$$

$$dy = du$$

$$\int \frac{8 du}{u^3} = \int 8 u^{-3} du$$

$$= \frac{8 u^{-2}}{-2} = -4 u^{-2} + C_1$$

$$= -4 (y-9)^{-2} + C$$

$$= \boxed{\frac{-4}{(y-9)^2} + C}$$

$$28. \int (y-1)^7 dy : \text{Let } u = y-1, du = dy$$

$$\int u^7 du = \frac{u^8}{8} + C,$$

$$\boxed{\frac{(y-1)^8}{8} + C}$$

$$29. \int \frac{(\ln x)^9}{x} dx : \text{Let } u = \ln x$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$dx = x du$$

$$\int u^9 \frac{1}{x} x du = \int u^9 du$$

$$= \frac{u^{10}}{10} + C,$$

$$= \boxed{\frac{(\ln x)^{10}}{10} + C}$$

$$30. \int \frac{19}{2+5y} dy : \text{Let } 2+5y = u$$

$$\frac{du}{dy} = 5 \Rightarrow dy = \frac{du}{5}$$

$$\int \frac{19}{u} \cdot \frac{du}{5} = \frac{19}{5} \int \frac{du}{u} = \frac{19}{5} \ln u$$

$$= \boxed{\frac{19}{5} \ln (2+5y) + C}$$